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THE ABSOLUTE SCALE OF STELLAR MAGNITUDES.

By R. H. Tucker.

All attempts to grade the stars according to their brightness are based upon the adoption of an arbitrary scale, the basis of which may be said to be the order of magnitudes first given to the brighter stars by the earliest naked-eye observers. The sixth magnitude is ordinarily taken to be the limit of naked-eye visibility, though for practiced observers, with keen vision, the seventh magnitude may be set as the limit. The ratio of brightness found to exist in the early estimates corresponds to the number whose logarithm is 0.4; that is, in passing to a brighter grade the ratio is approximately 2.5, and in passing to a fainter grade the inverse ratio is 0.4, in terms of the light.

Modern adjustments of the scale of magnitudes are usually based upon the adoption of the sixth magnitude from existing Uranometries, and with fairly close adherence to the ninth magnitude of Argelander, as given in the first *Durchmusterung* for the northern sky. Observers with telescopes have in general endeavored to establish a scale in conformity with this range of the so-called "historic scale."

It must be noted that, for the brightest stars, visual estimates made with a telescope are not likely to be as precise as those made with the naked eye. In any case, the distinction of the gradations of brightness towards the fainter end of the scale, visible under certain conditions, can be more precisely made. The faintest stars visible with the naked eye can be very accurately gauged; and, similarly, the faintest stars visible in a telescope should be classified most easily. The illumination of the field of view enters as a factor, and may be said to act as a photometric aid. Thus, in the telescopes of the usual type of meridian circle, stars from eighth to tenth magnitude can be quite precisely graded, while, without illumination, fainter stars would be visible, and would be easily classified. For stars between sixth and eighth magnitude, fairly close estimates may be expected; and, with considerable practice, the scale can be carried down to fourth magnitude. Beyond that point the lack of frequent observations of bright stars in ordinary observing experience, and the very noticeable variation in the quality of the images of bright stars, due to different conditions of steadiness and definition, make the estimates of the brightest stars with a telescope very uncertain.

In using the 36-inch telescope, for the purpose of establishing standards of visual magnitude for the faintest stars visible, in the early nineties, the range of magnitude from the fourteenth to the seventeenth inclusive could be quite closely estimated; while it would have required a great deal of experience with that instrument to estimate the relative grades of stars brighter than these classes. The same region had been previously charted with the Harvard College 15-inch and the Washington 26-inch, each telescope having added approximately one unit of magnitude at the fainter limit. The *Uranometries,* with naked-eye estimates, furnish the visual scale for the brightest stars; small telescopes carry the scale down to fainter stars, and the largest instruments are necessary for visual estimates of the faintest grades. The errors of absolute scale may be conceived as cumulative, in thus establishing the scale of visual magnitudes; but the limit of visibility with any size of telescope, used with dark field, is a check upon the faint end of the scale, and the faintest stars seen are very likely to be brighter than the theoretical limit of visibility.

The photometer, in various forms, is an instrumental device for extending the scale of magnitudes in both directions from the standard type, which is assumed to be the historic sixth magnitude of the visual scale. The results of measures with the photometer are based upon comparison with the visual estimates.

The process followed is either to equalize two sources of light, one the star whose brightness is to be measured, the other source being an artificial star, or a standard star, whose beams are reflected into the same field of view; or to determine the point at which the light of the real star is extinguished, and measure the extinction employed.

For varying the amount of light from either source, wedges of partially opaque glass can be used, sometimes darkened by the photographic process; or methods of reflecting a part, or all, of the light may be used.

While it is often contended that photometric work consists of measures in place of simple estimates, and that therefore the results should be more reliable, it should not be forgotten that the result of every measure depends, originally or finally, upon the human judgment in some form. In photometric work, the estimation of exact equality of two sources of light, or of entire extinction, may be more precise than any estimation of relative difference of light, but the element of personal judgment cannot be entirely eliminated. Photographic determinations of the brightness of the stars contain the personal element also, when the comparison of star images is made. Both these instrumental methods depend to some extent upon comparison with adopted visual estimates, of a part of the scale at least, for establishing the basic type, and for checking the theoretical variation of their measures with variation of type. It is possibly open to question whether the results of either form for the brightest stars are improvements upon the best visual naked-eye *Uranometry* magnitudes, such as those of GOULD in the southern hemisphere.

The extension of the scale to the fainter stars is, however, more in line with the purpose of this note.

In the endeavor to give an expression which will represent the actual distribution of the stars according to brightness, two assumptions are generally made at the start, neither of which is believed to be more than partially valid, for more than a limited part of the scale at most. These assumptions are: uniform distribution in space and equal intrinsic brightness. The first assumption is at once contradicted by the known clusters, the groups of stars, and the vast stream of the Milky Way. Still, by taking large areas of sky, the law of distribution can be tested, as far as it concerns the various types of brightness. The second assumption is contradicted by the fact that we know that there must be a large range of size among the stars from the study of their masses, and we also recognize from the sequence of the various spectral types that there is a process of evolution, by which the stars develop, and all degrees of brilliancy may be represented, from the newest stars to those partially burned out, or, in the case of the "dark stars," the light may have been totally extinguished.

The two assumptions would make the brightness the direct and only criterion of distance, while there are many cases in which faint stars are known to be nearer to us than some of the brighter stars. The expression based upon these two assumptions can be used as a test of the distribution, and its variation from counts of stars should show how far such assumptions may fall short of explaining actual conditions.

The expression for the summation of the number of stars to any order of brightness is usually put in the form—

$$\Sigma\, m = a\, b^{\mathrm{m}}$$

In this expression m is the limiting magnitude, a is the number of stars for unit radius, and b depends upon the assumptions first made. If rigorously true, b would equal $\delta^{3/2}$, when δ is the ratio of brightness for a difference of one magnitude; the volumes being proportional to the cubes of the radii of the enclosing spheres, and the brightness of the stars being inversely proportional to the squares of their distances.

The theoretical value of b would thus be the number whose logarithm is 0.6, or 3.98. It would then correspond to the ratio of volume for a decrease of one magnitude in the scale.

The law which is expressed by the above formula for the summation is known as that of the absolute scale of magnitude. Visual estimates have endeavored to conform to this law, in establishing the visual scale; photometric measures are based upon the same law; and photographic measures of star images are made to conform to the law, by comparison with the established scale. As counts and estimates of the actual numbers of faint stars become available, the value of b can be determined at various points in the scale, treating it as a variable quantity, to conform to the deviation of fact (real distribution) from theory (uniform distribution and equal brightness), by the original assumptions.

This is a convenient process also for the tests, since b can be determined at any point of the scale, by the comparison of the summations of two grades, leaving the quantity a invariable.

The values of b have been found to diminish, for the faintest grades of brightness. This would be anticipated, unless the total number of stars in the universe were assumed to be infinite, and also from the fact that the whole amount of light received from the stars does not increase indefinitely. The successive values of b for small differences of magnitude may be expected to vary slowly, and the summation may then be used to test the scale itself, over a limited range at least.

Some applications of the theoretical law to actual counts will be made here.

The northern *Durchmusterung* of ARGELANDER contains a practically complete list of the stars to ninth magnitude for one half of the sky, and the southern *Durchmusterung* provides, in its several parts, a similar list for the other half. A systematic count of the northern *Durchmusterung* was made by LITTROW, and published in the 1869 *Sitzungsberichte* of the Vienna Academy of Sciences, and it is printed in GOULD'S *Uranometry*, in his discussion of the distribution of the stars. The numerical expression for the summation of the faint stars, to 9.0, in the northern sky, has the following form:

$$\Sigma m = 0.5208\,(3.8685)^m.$$

It is the portion of the scale from 7.0 to 10 which has the most interest just here; with later its extension downwards beyond the limits of the visual *Durchmusterung*. For this reason, it has appeared worth while to give the count of ARGELANDER in detail for this portion, especially since a similar count is already on record, for a part of the southern sky, in the note printed in the *Astrophysical Journal* for May, 1898. The following table is taken from LITTROW'S count, and the summation has been made for the separate tenths of each magnitude:—

TABLE I—ARGELANDER'S NORTHERN *DURCHMUSTERUNG.*

Mag.	No.	Σ	Per 1000	Σ	*b*
Br.	....	5,876	..	19	3.866
7.0	2,141	8,017	7	26	3.965
7.1	345	8,362	1	27	3.911
7.2	984	9,346	3	30	3.898
7.3	1,356	10,702	4	34	3.907
7.4	516	11,218	2	36	3.851
7.5	2,860	14,078	9	45	3.899
7.6	609	14,687	2	47	3.851
7.7	1,537	16,224	5	52	3.834
7.8	2,484	18,708	8	60	3.837
7.9	991	19,699	3	63	3.798
8.0	5,622	25,321	18	81	3.853
8.1	1,778	27,099	6	87	3.822
8.2	3,650	30,749	11	98	3.818
8.3	4,609	35,358	15	113	3.822
8.4	3,101	38,459	10	123	3.798
8.5	9,788	48,247	31	154	3.840
8.6	4,189	52,436	13	167	3.817
8.7	6,799	59,235	22	189	3.812
8.8	10,963	70,198	35	224	3.827
8.9	7,596	77,794	24	248	3.813
9.0	23,277	101,071	74	322	3.869
9.1	15,615	116,686	49	371	3.872
9.2	20,734	137,420	66	437	3.884
9.3	31,278	168,698	99	536	3.913
9.4	34,951	203,649	111	647	3.935
9.5	111,276	314,925	353	1,000	[4.061]
		Σ	1,000	mean	3.858

In the above table the stars brighter than 7.0 have been placed in the first group. The irregular variation of the numbers, for successive tenths of the scale, is noticeable. This is due to the fact that ARGELANDER estimated to half magnitudes only, hence the 7.5 and 8.5, and the even units will naturally preponderate. Also it is evident that the mean of 7 and 7½ was rated as 7.3, in the published scale by tenths; the mean of 7½ and 8 was rated as 7.8, in tenths; and the same convention was adopted throughout. The other tenths of the scale are derived from three or more observations.

The average number, per thousand stars of the faintest grade inclusive, is given, in the table, for convenience of expressing the distribution in a form which is evident at a glance. Taking the adopted value of *a*, 0.5208, for the summation to 9.0 in the northern sky, the values of *b* have been computed for each tenth. The value for 9.5 shows at once the elastic nature of that grade; it was intended in the original estimates to include 9½ and fainter stars, and the computation of the individual values of *b* shows that the mean, 3.86, corresponds to the summation to a grade between 9.8 and 9.9 on the same scale of magnitudes. To give the number equivalent to the summation to that point, stars as faint as 10 were probably observed. All of the grades, by tenths, fainter than 9, are somewhat in excess, also, and it is evident that the observed scale changed slightly at that point, and that stars of the fainter classes were generally estimated too bright.

The number of stars included in the grade 9.0 is in excess, apparently, by about 8,000, as compared with the grades each side; and the numbers at the other even units are in similar proportional excess. ARGELANDER expressed the opinion that the stars of ninth grade were probably too numerous, in his classification; but the test supplied by the table indicates a fairly consistent scale to ninth magnitude.

The results of a count of magnitudes, by tenths, in the first ten degrees of the Córdoba *Durchmusterung*, are given in the following table, as they were originally published in the *Astrophysical Journal*. Some 37,000 stars were included in that count. The tabulation of stars per thousand is made in the same form as that used above for the northern region. The lowest grade, 10, at Córdoba was an elastic one, and included many stars that were consciously estimated to be much fainter than tenth magnitude. This was necessary, to some extent, in order to be sure of including all stars of tenth magnitude. The numbers at the half magnitudes and even units of the scale are generally somewhat in excess. The progression in the other tenths is nearly uniform, since the original estimates by quarters include more than two observations for a large proportion of the stars. The average per thousand refers, of course, to the total summation.

TABLE II—FIRST TEN DEGREES OF CÓRDOBA D. M.

Mag.	Per 1000	Σ	*b*	Mag.	Per 1000	Σ	*b*
Br.	..	6	3.88	8.5	10	57	3.95
7.0	2	8	3.96	8.6	8	65	3.95
7.1	1	9	3.93	8.7	11	76	3.96
7.2	1	10	3.95	8.8	10	86	3.95
7.3	1	11	3.95	8.9	11	97	3.95
7.4	1	12	3.91	9.0	24	121	3.98
7.5	3	15	3.95	9.1	23	144	4.00
7.6	1	16	3.93	9.2	26	170	4.01
7.7	3	19	3.94	9.3	27	197	4.01
7.8	2	21	3.93	9.4	33	230	4.02
7.9	2	23	3.90	9.5	54	284	4.05
8.0	5	28	3.93	9.6	63	347	4.07
8.1	4	32	3.93	9.7	87	434	4.11
8.2	5	37	3.93	9.8	100	534	4.14
8.3	6	43	3.94	9.9	86	620	4.14
8.4	4	47	3.92	10.0	380	1,000	[4.28]

The values of *b* in the table are fairly uniform and constant from seventh to ninth magnitude, at which point they begin to increase perceptibly. This indicates that the scale of the estimates begins to change at ninth magnitude, the faint stars being estimated too bright. ARGELANDER'S scale indicates a similar change, beginning at 9¼. In either case the increase could be explained by an increase in the actual numbers of faint stars; but such an assumption is far less probably true than changes in the personal estimates. The 10 of Córdoba, understood to be an elastic grade, includes the equivalent of the total number of stars to 10.5 of its own scale, if we adopt the value of *b* at 9.0 of that scale. If, however, we allow for the increase in *b* from 9 to 9.9, then the 10 of the catalogue includes a number equivalent to the summation to 10.2 of this slightly varying scale.

Unless there are actually numbers of faint stars between ninth and tenth magnitude in excess of the proportion of brighter stars, each of the two scales, from ninth magnitude to the fainter limit, can be corrected on the basis of the summation at ninth magnitude by the summation at the individual tenths, and this has been done below. The correction to ARGELANDER'S scale does not become important until 9.3 is reached. The faintest grade, 9.5, would be below 9.8 on its own scale.

The correction to the Córdoba scale becomes sensible at 9.5, amounts to two tenths at 9.8, and the correction to the ten of the scale is probably the same amount. The 10 of the catalogue, however, is equivalent to the inclusion of stars to 10.4. The Córdoba estimates by the comparison of numbers, are two tenths brighter than ARGELANDER, at ninth magnitude; and this difference has been applied to derive the equivalent of the corrected ARGELANDER scale, in the last column, for the Córdoba estimates. The comparison of the total numbers of stars in the two catalogues indicates an extension of the Córdoba scale to 0.9 magnitude beyond the limit of ARGELANDER; and the faint 10 of Córdoba thus corresponds to the true 10.7 of ARGELANDER. The 9.5 and 9.6 of Córdoba are probably brighter, on the average, than the 9.5 of ARGELANDER, which is nearly the same as the 9.7 of Córdoba.

TABLE III—CORRECTED MAGNITUDE SCALE OF FAINT STARS.

Argelander Mag.	Nor. D. M. Corrected.	Córdoba — 32° Corrected.	Reduced to Argelander.
9.1	9.10	9.12	9.3
9.2	9.22	9.23	9.4
9.3	9.37	9.33	9.5
9.4	9.51	9.43	9.6
9.5	[9.82]	9.58	9.8
9.6	...	9.71	9.9
9.7	...	9.86	10.1
9.8	...	10.00	10.2
9.9	...	10.10	10.3
10.0	...	[10.43]	[10.6]

In the discussion of the photometric scale of magnitudes, in a recent volume of the publications of the Harvard College Observatory, *Harvard Annals,* **72**, page 84, there is a reference to the summation of the Córdoba *Durchmusterung* for the zone —49°. On the basis of the number of stars in this zone of one degree, the total number of stars in the sky down to the same limit of brightness is found to be 2,533,000. This is an average of sixty-one stars to a square degree; while the first two Córdoba volumes, for the region —22° to —42°, give an average of fifty-five stars, which would be equivalent to 2,269,000 stars for the area of the whole sky. The increase in number in this case does not necessarily imply a definite extension of the magnitude scale, which was intended to be complete to the tenth magnitude only, but probably a slightly increased proportion of faint stars, below the limit of completeness.

This enumeration was compared with the numbers for the photometric scale, published in *Harvard Annals*, **48**, Table XXI, page 178, and the conclusion was drawn that the Córdoba scale includes an equivalent of the photometric scale to the twelfth magnitude. There is evidently a large difference between the respective scales, for the faint stars.

The photometric scale given in Table XXI, Volume 48, was based upon counts and estimates of stars, of various degrees of brightness, down to the magnitude 13.5. The counts were plotted, and a curve drawn, from which the figures corresponding to distribution were taken. Reference to the original volume should be made for the details of the process. The method is based upon the law of absolute magnitudes; and in the expression $\Sigma m = a\, b^{m}$, the a is determined from a comparison of the counts with the respective volumes, and is then adopted as a constant 3.7. The b is taken as variable, its value being derived from the smoothed curve, and ranging from 3.34 at magnitude 0, to 2.95 at 13.0, when computed from the respective summations at these points of the scale. The values of b are tabulated below:—

TABLE IV—PHOTOMETRIC SCALE.
(*Harvard Annals*, 48, Table XXI.)

Mag.	Stars.	*b*
7.0	14,960	3.27
8.0	45,710	3.26
9.0	134,300	3.21
10.0	373,300	3.16
11.0	957,200	3.11
12.0	2,239,000	3.03
13.0	4,786,000	2.95

The values of b in the sequences in this table are naturally smoothed out by the process used in forming the numbers. The numbers of stars have been computed for the whole sky, hence, in making comparisons with those of ARGELANDER, the figures of the latter must be doubled. The number of ninth magnitude stars in the sky by ARGELANDER's scale would be 202,000, and, by the Córdoba scale, 275,000; hence the photometric summation at ninth magnitude has only two thirds of ARGELANDER's enumeration, and one half that of Córdoba (first ten degrees).

There is some uncertainty in the direct comparison of the numbers, since the photometric scale is summed up to half magnitudes, and the visual scales to tenths. According to a note from Professor PICKERING, of date March 10, 1913, the tabulation to ninth magnitude includes the stars up to that grade, exactly; hence, to make this enumeration correspond to the visual scale, a number equivalent to the half of the number of stars per tenth magnitude of the latter should be added to the photometric enumeration. This would be about 8,000 at ninth magnitude, a small percentage.

Since the relative brightness of the various scales at ninth magnitude is a most important basis of comparison, the numbers from the two remaining parts of the *Durchmusterung* should be considered here. The *Cape Photographic Durchmusterung* contains about 455,000 real stars in the area from — 19° to the south pole, which is about 67 per cent of the southern hemisphere. Of these stars, 81,481 are given as magnitude 9.0 or brighter, about 18 per cent. This enumeration is equivalent to 243,000 ninth magnitude in the whole sky, or about 1,350,000 stars down to the lowest grades included in the photographic catalogue.

There is a rather sharp division in the catalogue at —34°, which I discussed, to some extent, in the note in the *Astrophysical Journal*. The percentage of ninth magnitude stars drops from 23 per cent north of —34° to 16 per cent south of that point. The lower limit of the catalogue was extended a full quarter of a unit of magnitude, but the estimated ninth magnitude type has apparently remained the same. The limiting faint type of star has been raised, in total number for the sky, from one million to one and a half million, by this change in extension.

In SCHÖNFELD's southern *Durchmusterung* of 133,000 stars, in the area —2° to —22° inclusive, 34,119 stars are of 9.0 and brighter, according to count by SEELIGER,[1] or 25.7 per cent.

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[1] *Cape Annals*, IX, page 185 B.

The equivalent of his full catalogue would be 747,000 stars for the whole sky, or 192,000 stars to 9.0 inclusive. This last number is slightly smaller than ARGELANDER'S summation, and much below the Cape Photographic and Córdoba first ten degrees. SCHÖNFELD'S summation at 10 and 9.0 would give the value of b 3.898, which indicates that his scale is probably consistent throughout, to the lowest grade.

With the value $b = 3.9$, the photographic limits would be 10.08 and 10.34, above and below — 34° declination, on its own scale. Its catalogued stars vary from 9.6 to 11.4 at the lower limits, for various parts of the sky. The feature of the photographic catalogue which appears most difficult to understand is the small numbers and wide range of the faintest stars recorded. It would be expected that the photograph would give a very definite limit of brightness, at which the faintest grade would be very nearly complete.

It remains to translate the four scales into the photometric scale, according to the respective enumerations. The basis is table XXI, *Harvard Annals,* **48,** page 178, already referred to above.

ARGELANDER and Córdoba are given in detail, for the fainter grades, for which counts and close estimates of numbers exist.

TABLE V—PHOTOMETRIC MAGNITUDES OF D. M. SCALES.

Mag.	Argelander.	Córdoba.	Schönfeld.	Cape.
7.0	7.06	7.17	...	...
8.0	8.09	8.30	...	...
9.0	9.39	9.69	9.34	9.58
9.1	9.54	9.86	...	...
9.2	9.69	10.03	...	...
9.3	9.90	10.18	...	...
9.4	10.09	10.34	...	...
9.5	[10.53]	10.71	...	...
9.6	...	10.78	...	...
9.7	...	11.03	...	...
9.8	10.5	11.26	...	...
9.9	...	11.43	...	...
10.	...	[12.02]	10.70	
10.1	...	...	...	11.1
10.2	...	...	...	...
10.3	...	...	...	11.5
10.4	...	12.0	...	...

From the above photometric basis of enumeration, Córdoba estimates would appear to be one tenth brighter than ARGELANDER at seventh magnitude; two tenths at eight; three tenths at ninth; and three quarters of a magnitude brighter at the faint limit of ARGELANDER, which is 9.8 of its own scale.

SCHÖNFELD and ARGELANDER are in close agreement at nine and ten of both scales.

The Cape Photographic estimates are one tenth fainter than Córdoba at ninth magnitude, and its faintest grades correspond nearly to Córdoba 9¾ and 10, above and below —34° declination, respectively, or about three tenths fainter than the Córdoba scale.

ARGELANDER'S scale gives estimates one tenth brighter than the photometric at eighth magnitude; four tenths brighter at ninth; increasing to seven tenths brighter, for the faintest stars of the northern *Durchmusterung*.

As a further test of the difference between the photometric and visual scales, the following comparison has been tabulated from the separate results recently published in *Harvard Annals*, 72, which includes estimates made by Professor BAILEY at the time of the photometric measures. These last estimates are individually, and collectively, very uniformly consistent with the Córdoba magnitudes, with a slowly progressive increase of difference. The average residual appears to be smaller than would be encountered among the separate independent estimates of the observation made for the *Durchmusterung*. The comparison extends through the whole circuit of right ascension, for parts of the zone —29° 50′ to —30° 0′. Only 1,133 of the *Durchmusterung* stars were measured, while the 10′ belt contains about 3,000 stars, and some of the brighter stars lie outside the belt.

The differences of the table are given for quarter magnitudes, three individual tenths being combined, and the unit magnitudes being given separately.

As before noted, the range of the photometric scale is nearly double that of the visual, between these limits, 3.8 units against Córdoba two units; or 50 per cent larger, if the Córdoba ten be assumed to extend to 10.4 of its own scale.

TABLE VI—DIRECT COMPARISON OF PHOTOMETRIC AND CÓRDOBA VISUAL SCALES.

D. M. Mag.	No.	Phot. Mag.	Δ	Bailey.	Δ
Br.	43	...	+0.18	...	+0.19
8.0	31	8.26	+0.26	8.34	+0.34
8.1	9	8.45	......	8.41	......
8.2	11	8.55	+0.37	8.65	+0.37
8.3	23	8.72	......	8.64	......
8.4	11	8.64	......	8.82	......
8.5	23	8.82	+0.39	8.89	+0.43
8.6	19	9.22	......	9.07	......
8.7	30	9.27	......	9.10	......
8.8	20	9.36	+0.57	9.15	+0.37
8.9	20	9.49	......	9.25	......
9.0	64	9.74	+0.74	9.46	+0.46
9.1	54	9.97	......	9.54	......
9.2	43	10.27	+1.02	9.69	+0.46
9.3	58	10.43	......	9.74	......
9.4	58	10.58	......	9.85	......
9.5	55	11.00	+1.41	9.98	+0.49
9.6	86	11.14	......	10.15	......
9.7	98	11.49	......	10.31	......
9.8	55	11.61	+1.82	10.47	+0.63
9.9	46	11.76	......	10.52	......
10.0	276	12.08	+2.08	10.76	+0.76

The estimates of BAILEY are close to the photometric scale at the bright end, but are 1.3 magnitude brighter at the faint end.

The direct comparison shows that the photometric measures are much fainter than the visual estimates and the photographic measures, for all the fainter grades of stars. The comparison of the respective summations of stars included gives the same indication. In the summations, the values of b in the photometric scale are smaller than generally accepted for the so-called absolute scale of magnitudes. On the other hand the value of a is large; that adopted in the formula for the northern *Durchmusterung* being only slightly in excess of unity (1.04) for the whole sky, or less than one third of the value derived for the photometric. No account has been made here of the possible loss of light by an absorbing medium in space. In general, the decrease in b indicates an approach to the limit, not only of the stars visible, but of actually existing stars.

May 13, 1913.